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REVIEWS

Elongational Flows: Aspects of the Behavior of Model Viscoelastic Fluid.

By C. J. S. PETRIE. Pitman, 1979. 254 pp. $17.50.

In contrast to the well-charted realm of Newtonian fluid, the mechanics of non-Newtonian fluids is a poorly explored and undeveloped territory, to a large extent still shrouded in the mists of indefinite constitutive laws. It is with great interest, then, that we should welcome Petrie's work, coming as it does as a kind of progress report from a newly established outpost on this scientific frontier.

While the past twenty-five years have seen impressive activity in the rheology of nonlinear viscoelastic fluids, particularly polymer fluids, most of the interest in elongational flows is relatively recent. The interest in these flows, sometimes called extensional flows or motions (terms preferred by the reviewer), stems principally from two facts. First of all, they represent in effect the simplest type of material deformations, involving as they do materially invariant principal axes of stretch or extension. That is, the principal directions of stretch and rate-of-deformation at a material point are always embedded in the same material lines. Roughly speaking, this affords the highest degree of deformational symmetry and as a result the minimum number of material functions necessary to describe the resultant stress in an isotropic material.

Rheology, like other branches of physics, tends to seek out such symmetry whenever confronted with unknown or incomplete quantitative laws of physical change, and extensional flows have thus to come be viewed as a potentially attractive means for investigating rheological behaviour as well as for testing constitutive equations. Furthermore, the important class of *materially steady* extensions is known to involve exponentially large material strains, which sets them off from the more accessible simple shear or viscometric flows of classical rheology. As generally recognized for several years now, steady extensions and other varieties of so-called 'strong flows' serve generally to divide mathematical models for fluid rheology into two, rather distinct classes: those which exhibit a solid-like unbounded growth of stress at certain critical rates of extension, and those which do not. As it turns out, this very nearly allows one to divide rheologists into two corresponding classes: those who accept and even embrace such fluid models, and those who do not. I think it proper to alert the reader at this point that the above author and this reviewer tend to fall into different classes, as later remarks here will serve to indicate.

A second reason for studying extensional flows is their direct relevance to a number of technically important processes, such as fibre spinning and film forming in the polymer industry. Here, a complete fluid mechanics, that is the rheology *plus* the dynamics, is very much wanted.

Petrie's treatise provides an exhaustive and up-to-date survey of the existing research literature on extensional flows, which despite his prefatory disclaimer, can be viewed as much a monograph as a 'research note'. The focus of the book is on the kinematics and continuum rheology of various types of extensional flow, together with a rudimentary treatment of the mechanics of isothermal filament spinning. Although he touches from time to time on molecular theory and enters somewhat into

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general continuum mechanics, the author's intent is clearly to base his treatment and, it might be added, many of his conclusions on a few, relatively simple differential models of fluid rheology. In this elementary approach lie several strengths as well as weaknesses of the book.

On the one hand, the use of elementary rheological models coupled with the author's rather casual style of discourse makes for easy reading and exposes a number of fundamental issues which are as well conveyed by simple mathematical models as by the more complex. On the other hand, there is a notable lack of economy in the treatment (compare the 254 pages with the 130 pages on viscometric flows in the classic 1966 Springer Tract by Coleman, Markovitz and Noll) and a certain loss of rigour in the author's methods and conclusions.

Chapter 1 is an introductory historical and phenomenological survey. The author perhaps wisely avoids a technical definition of 'elongational flows' in the Introduction, but, as far as I can ascertain, unfortunately *never* states a precise definition anywhere. Thus, despite the considerable elaboration on kinematics in chapter 4 (including an incompletely proved conjecture in page 131 whose proof follows rather directly from known properties of commutative matrix algebras), a precise definition of 'elongational flow' is not to be found in the very chapter where one might most expect it. Also, while he introduces the two customary dimensionless time ratios, the Deborah number and the Weissenberg number, it is never convincingly demonstrated that they should be treated as distinct, and this never becomes evident in practice since these parameters are not used to much advantage in the subsequent discussion of rheology and kinematics of later chapters.

Chapter 2 is an elementary discussion of kinematics and a survey of special constitutive models for elasticoviscous liquids, with an almost immediate specialization to a class of differential models which can be derived from a type of 'generalized Jeffreys model'.

There is a peculiar quality to the discussion of chapter 2, as indeed for other parts of the book, which makes it sometimes difficult to know whether the author is talking about kinematics of about dynamics. Thus, after covering uniaxial elongation in §2.3.1 he basically discusses it one again as 'spinning' in §2.3.1. Also, for some unexplained reason he lumps 'equal biaxial stretching' together with 'pure shear' or, as he calls it, 'strip biaxial extension'. They are not kinematically similar nor, as the author notes elsewhere (p. 37), are they rheologically related for the more realistic fluid models. The author's adoption of the term 'strip biaxial' is as unfortunate as the traditional 'pure shear' for designating *planar extension*. Unless rigidly mandated by some misguided convention, one would hope that we might agree as scientists to use the terms 'uniaxial', biaxial' and, hence, 'triaxial' in a reasonably consistent manner. Also, as a minor point, it might be observed that the definition of 'extra stress' on p. 20 as that 'given by the constitutive equation' will probably not stand the test of time.

Chapter 3 provides a useful summary of experimental techniques and associated problems but, contrary to the implication of the title, places little emphasis on quantitative assessment of experimental *results*. A valuable discussion is presented of the difficulties in interpreting data from filament-spinning experiments, where 'initial conditions' or upstream prehistory together with convective unsteadiness militates against obtaining a unique material function. This reviewer was mildly surprised to find a cautiously stated query into this matter by himself and a co-worker (in reference W8) characterized by Petrie as an idea wanting to be 'sustained' (p. 100). It was all the more disappointing not to find this discussed later, at greater length, as the remarks on p. 101 seem to promise.

In chapter 4, entitled 'Flow classification', the author returns to the subject of kinematics, where he provides an engaging discussion of several elementary flow fields which involve extensional deformations. In the word 'flow', just employed here, the vicissitudes of language become apparent. So do they also in the author's discussion of kinematics, where the work 'steady' is a major culprit, as much through its unqualified presence as through its unexcused absence. Thus, the author's characterizations of shear and elongation as either 'strong' or 'weak' surely have to be qualified by the adjective 'steady'. Also, this same adjective is often used without much regard as to whether one is speaking of *spatial* or *material* steadiness. On the obverse side, the author's distinctions between spatially homogeneous and non-homogeneous flows appear, for the most part, to be superfluous. Spatial non-homogeneity like spatial unsteadiness has, as we know, only indirect rheological significance, at least for the simple (Noll) materials discussed in this treatise.

On a related point, the author's adherence to the Cartesian form of the velocity field for a homogeneous extension leads to an evident misinterpretation of a result of Coleman and an erroneous expression in equation (44.1), for the velocity gradient.

The final chapters, 5–7, are devoted largely to uniaxial extension or, as the reviewer would call it, 'simple' extension. The author now relies heavily on the term 'stretching' to designate his kinematics. Also, he imposes a needless restriction to homogeneous extensions which at the same time are allowed to be materially unsteady. Much of chapter 5 is based on the consideration of the detailed simple-extensional response of the elementary Jeffreys fluid, and variations thereof. The intent clearly is to forego the memory-integral form of such fluid models, which give stress explicitly in terms of deformation history on $(-\infty, t)$, in favour of a differential form aimed at later application in chapter 6 to spinning. This requires the derivation of appropriate 'initial condition' to be used on $(0, t)$. With the type of elementary viscoelastic fluid models relied on in the present treatise, this kind of mathematical trade-off is a well recognized possibility for even more general flow problems. As a benefit, it gives dynamical equations which are explicit in the velocity field and which, for the case of steady spinning of a threadline, reduce to nonlinear ordinary differential equations under various simplifying assumptions. In taking this approach, the author respects the conventions of much of the recent literature on spin-line dynamics, to which he has made several contributions, and he is thereby able to sketch the problem in a mathematical medium which will appeal to many applied mathematicians and engineers. On the other hand, he largely glosses over the point that, for memory fluids, the essence of the spinning problem, indeed of more complex flow problems, can generally be captured *only* in the notion of *functional-differential* equations or, more concretely, *integro-differential* equations. Recent history suggests that even the most practical-minded will eventually accept the latter view, especially when it is implemented numerically on a digital computer!

Despite the occasional theoretical lapses, including the conjecture of a new principle on p. 153 that looks suspiciously like the fading-memory principle, chapter 5 contains a number of valuable insights, which are highly recommended to any who propose to work in this area. The same can be said of chapter 6 which summarizes much that is known about the modelling, dynamics and stability of viscoelastic spin lines.

At this point in the history of the subject, the reviewer would readily concede the wisdom of the author's basing his research note almost entirely on the more tractable rheological models. Within that framework, however, the reviewer cannot agree with the summary assessment of the 'corotational' model on p. 161. It appears that the author's opinion may be based largely on the fact that the simple corotational Maxwell model exhibits a kind of 'necking' failure in extensional flow, which he discusses on pp. 63 and 165 ff. Thus, as signalled by my introductry comments above, the author appears to join that school which rejects this type of behaviour yet accepts the *infinite forces* that arise from other elementary forms of the Maxwell fluid, as exemplified by the results displayed in figure 2.9 on p. 68. It is to be hoped that the door has not been closed on this long-standing question, since it pertains to possibly interesting physical phenomena and to the adequacy of current rheological models in representing them, especially the simple type of models relied on in the present work.

In concluding here, Petrie's treatise is recommended highly to those wishing to be brought up to date on the activity in this important area of research, and especially to those having a confident grasp of modern rheology. Perhaps many of the reservations expressed above in this review can be best summarized, in something of a counterpoint to the author's epigrammatic citations of Borges, by acknowledging our legacy from Jeffreys and the ultimate master: 'When *I* use a word', Humpty Dumpty said, in a rather scornful tone, 'it means just what I choose it to mean – neither more nor less' (Lewis Carroll, *Alice in Wonderland*).

J. D. GODDARD

Magnetohydrodynamic Flow in Ducts. By H. BRANOVER. Wiley and Israel Universities Press, 1978. 290 pp. £21.25.

There are not yet so many books on engineering magnetohydrodynamics that another is unwelcome, particularly a book like this that fills an unoccupied niche in the Western Literature. The attractions of Branover's book are that is it an *experimentalist's* book (by an experimentalist, for experimentalists), a useful antidote to the many theoretical accounts of the subject that are available. Also it makes more readily available to Western readers some of the extensive work on liquid-metal MHD that has been done in the Soviet Union, notably at the Institute of Physics in Riga, where the author worked before his move to Israel. In these circumstances he and his linguistic advisers are to be congratulated on the general quality of their English prose.

As the title indicates, the book concentrates on duct flow, which is that part of liquid metal MHD which, ever since Hartmann's classic work, has received most, and perhaps excessive attention. If it is true that current interest in engineering MHD is generally broadening from duct flows into the electrically driven, closed, circulating motions characteristic of MHD in the context of industrial metallurgy, then Branover's book is timely in that it sums up the state of play in duct flow research at a slackening of the action. The book is aimed primarily at the research worker and, despite the author's claim in the preface, is not in my view appropriate for a student wholly new to the subject.

After a readable introductory, descriptive chapter, chapter 2 states and discusses the governing equations of MHD and the main effects and dimensionless parameters. I fear that this chapter would be confusing to the uninitiated reader, as the order and selection of topics seems somewhat quixotic and the development of the ideas leaves something to be desired. There are errors of fact, such as the statement (on p. 16) that the electric field strength is uniform at the surface of a perfect conductor (although this *is* true in the case being considered). Nor is it necessary (p. 20) to assume in MHD that the dielectric constant is close to unity. There are minor irritations, such as the change to the symbol b for magnetic field without warning. In chapter 1 the significance of MHD for aerospace technology is greatly overstated.

It is in chapter 3 that the author begins to show his paces. Here we have a most valuable compilation of wisdom for the edification of would-be practitioners of experimental liquid-metal MHD research. In keeping with the title of the book, most of the emphasis is on duct-flow work but some information on alternative forms of experiment is given. Due attention is given to instrumentation appropriate to liquid metal flows. The opacity that rules out visual methods is to some extent compensated for by the possibility of making electrical measurements, although relating these to the velocities is not always a straightforward matter, and so hot-film anemometry has its attractions. A copious bibliography is provided.

Chapter 4, on fully-developed duct flow, reveals again the merits of having an author who is an experimentalist, in touch with engineering realities. Theoreticians tend to forget that, as experimental or industrial flow circuits are rarely immersed *wholly* in a magnetic field, most of the flow is likely to be turbulent, however, in the test sections or useful devices where the field prevails the flow, if it is laminar, it is actually *laminarized*, as Branover puts it, perhaps incompletely in the length available. Chapter 4 is a useful summary of the results of the many workers in this part of the subject. In most cases the details of theoretical analysis are not given. Wherever possible comparison with experiment is made, with healthy exposure and discussion of the discrepancies. The reader has to be vigilant in the face of errors (e.g. 'conducting' and 'non-conducting' interchanged, etc.) and long suffering in the face of some obscurities (e.g. incompletely labelled graphs).

Chapter 5, on duct flow with streamwise variation of fields or cross-sections, is a healthy corrective to the over-exaggerated attention that has been given to the fully developed case. The emphasis of the Riga experiments is nicely complementary to the mainly theoretical work done in the West. We hear much about the so-called M-profile, in which the velocity is faster near the walls than in midstream. This is not a rare phenomenon, for it can occur owing to a variety of reasons. Branover so emphasizes it that the reader might be left with the impression that M-profiles are universal! He makes the important point that many published experiments, allegedly on fully developed flow, must have been dominated by the long-lasting disturbances (including M-profiles) introduced at entry to the magnet. The reader should note that Branover uses the term 'stabilization' here to mean the settling process towards the fully-developed state.

Chapters 6, 7 and 8 all relate mainly to the still controversial problems of turbulent duct flow. I recollect an earlier review I wrote for the *Journal of Fluid Mechanics* (vol. 10, 1961, p. 158) that began with a call for 'a strong lad' to undertake more experiments in the area. The review discussed Lawson Harris's valiant attempt to develop a semi-empirical theory from the very small body of experimental data then available. Whether or not it is because of that plea, the situation is now greatly different. Branover is to be congratulated on bringing together and correlating the large number of experimental results which are available, some collected strictly in duct flow and others in ingenious apparatus such as the moving grid experiment of Moreau and Alemany at Grenoble. The area is controversial and no doubt some of Branover's comments or deductions from the data will be questioned in various quarters, but he poses many of the outstanding questions and recognizes that definitive answers must still await more research, theoretical or experimental. Is it ungracious to wish that he had given more positive recommendations as to where precisely these next investigations should be concentrated?

MHD turbulence offers insights into ordinary turbulence, because of the diagnostic tool which the magnetic field provides in allowing new kinds of measurements and in providing controlled anistropy. This is of course seen in its ultimate form in the cases where two-dimensional turbulence, with its very degenerate kinematics, is closely approached. For this reason these later chapters would repay selective study by workers in turbulence generally.

As in the earlier chapters, the material is not uniformly penetrating or lucid. Chapter 6, on laminar instability and turbulence, for instance, begins with a very generalized discussion of quasi-isotropic MHD turbulence and its length scales which I found very diffuse. This chapter incidentally also covers the case where the field is parallel to the mean flow and where the effect of the field on turbulence is uncomplicated by any direct ('Hartmann') effect on the mean flow. The experiments here reveal interesting anomalies, with the flow appearing to settle into an incompletely laminarized regime. A feature of some MHD duct flows in transverse fields is that virtually laminar and turbulent flow can coexist side by side at high Hartmann numbers. The high-speed or high-shear layers that laminar theory predicts can apparently go turbulent with impunity. This is an interesting theme that Branover neglects, perhaps because there is not yet much experimental data on it.

Chapter 7 reveals comprehensively how the state of semi-empirical theories for the overall behaviour of duct flows has advanced considerably since Harris' book, mentioned above. As a bystander one wonders, on seeing the sophistication of some of the calculations, with their several unknown constants and functions, whether the available experimental data is capable of discriminating conclusively between rival approaches; and whether any of these theories are capable of predicting the overall character of turbulent MHD flow in any hitherto unexplored, novel flow configuration. To do so is surely the point of the exercise. It would have been interesting to have the author's views on the desirability of experiments on wholly different turbulent flows (e.g. electrically driven ones in closed containers) in order to shine a critical spotlight on the semi-empirical theories and to discern how soundly rooted in true physical mechanisms they are.

Finally chapter 8 turns to the question of how the imposed magnetic field affects the turbulence in detail, creating anisotropy and adding ohmic dissipation. Branover reviews the surprisingly extensive literature that reports measurements of local, fluctuating quantities. A dominant theme is the observation of quite strong and generally two-dimensional fluctuations in flows which, from their overall characteristics, would have been taken to be completely laminarized. The book in fact closes with a long discussion of the evidence for the two rival views that this two-dimensional turbulence is the residue of upstream three-dimensional turbulence or of two-dimensional disturbances at entry.

Though I have referred to some of its shortcomings, as well as its strengths, I am in no doubt that Branover's monograph deserves a prominent place on the shelf of all experimentalists in liquid metal MHD and that the wider community interested in turbulence, with or without magnetic fields, will also find much of interest in the book. It provides a conveniently accessible display of the current state of empirical research in MHD duct flow, both laminar and turbulent, a scene which still contains plenty of puzzles to intrigue the discerning reader and inspire further investigations.

J. A. SHERCLIFF

SHORTER NOTICES

Flow-induced crystallization in polymer systems. Edited by R. L. MILLER. Midland Macromolecular Monographs, Volume 6. Gordon & Breach, 1979. 370 pp. £23.60.

When a polymer melt or polymer solution flows, the polymer chains become preferentially aligned. The consequential reduction in entropy enables the polymers to crystallize at higher temperatures than in a quiescent melt or solution. In some flow processes the crystals so formed can be an undesirable inhomogeneity which is very sensitive to temperature. On the other hand, these crystals are usually in the form of fibres with a high degree of chain orientation which makes them very strong, theoretically stronger than steel weight-for-weight. This proceedings of a symposium held in August 1977 at the Midland Macromolecular Institute contains seven reviews and nine contributed papers of high standard. The progress reported is mainly experimental supported by some thermodynamic theory which relies heavily on information of the observed crystal morphology.

Structure and Mechanisms of Turbulence. I. Lecture Notes in Physics, no. 75. Edited by H. FIELDLER. Proceedings of the Symposium on Turbulence held at the Technische Universität Berlin, 1–5 August 1977. Springer, 1978. 295 pp. $14.80 (paperback).

This symposium's programme was devised so as to cover, in the words of the editor, essentially all problems and trends of present day's basic turbulence research. The subject matter of the four survey lectures, 46 papers and 11 short contributions concentrated on the structure of turbulent flows and related problems of scalar transport and noise. The symposium organizers excluded papers on prediction methods and modelling of turbulent flows, and so, unlike some conference proceedings, these are reasonably homogeneous (although the turbulent flows considered here are mainly inhomogeneous!). There are many fascinating photographs of flow visualization experiments and intriguing diagrams from computer calculation of vortex patterns. The review lectures are certainly provocative and readers of this

Journal will find plenty to interest them here and they or their library ought to obtain a copy.

Recent Developments in Theoretical and Experimental Fluid Mechanics. Edited by U. MÜLLER, K. G. ROESNER and B. SCHMIDT. Springer, 1979. 642 pp.

This book contains 62 separate articles of different kinds, including original papers, brief surveys, and historical notes, without a common theme or scientific purpose. Some are interesting, and some (inevitably) are not; and all of them are likely soon to be lost sight of, in an occasional publication of this type. The range of topics is wide, although transonic flow (114 pages) and hydrodynamic stability (132 pages) are prominent. The authors were invited to contribute to a volume which would commemorate the 50th birthday of Professor J. Zierep, Head of the Institute of Fluid Flow and Fluid Machinery at the University of Karlsruhe, and there is a prefatory eulogy by U. Müller. To the cool Anglo-Saxon eye, easily embarrassed by such demonstrations, 50 does seem rather early for a festschrift; can colleagues really 'express gratitude and appreciation' (as the preface says) to a person of this age and retain the freedom to differ from him and to give him helpful criticism in the way that is customary in the academic world?

Thermomechanics of Magnetic Fluids. Edited by B. BERKOVSKY. Hemisphere Publishing Co., 1978. 318 pp. $39.75.

During the last 15 years there has been a considerable growth in interest in magnetic colloids, that is, suspensions of sub-micron-size magnetized particles in fluid. Ferrofluids consisting of ferrite particles and a carrier fluid such as kerosene or water are now available commercially, and are used in a variety of applications, including rotary seals and bearings, controllable lubricants, damping systems, and ferrohydrodynamic pumping. The dynamical properties of magnetic fluids and the way in which a magnetic fluid interacts with an external magnetic field are intriguing, and are not yet fully understood. In October 1977 an 'Advanced Course and Workshop' on the thermomechanics of magnetic fluids was held at Udine, Italy, and this book contains the 18 expository papers delivered on that occasion. The meeting attracted a number of the leading workers in the field, and the book provides a valuable record of the present state of its development. A report on the meeting itself has already been published (*J. Fluid Mech.* vol. **87**, 1978, pp. 521–531).

Turbulence (Pipe Flows). By R. S. SRIVASTAVA. Indian National Science Academy, New Delhi, 1977. 64 pp.

This is a shorter book of 64 pages designed to provide a 'connected account and discussion on turbulent flow through pipes'. Inevitably the material reflects the author's own interests, which include flows in curved pipes.

Annual Review of Fluid Mechanics. Volume 11. Edited by M. VAN DYKE, J. V. WEHAUSEN and J. L. LUMLEY. Annual Reviews Inc., 1979. 556 pp. $17.50.

This year's volume contains the following articles.

The Kármán years at Galcit, W. R. Sears & M. R. Sears.

Model equations of nonlinear acoustics, D. G. Crighton.

Cavitation in bearings, D. Dowson & C. M. Taylor.
Self-sustained oscillations of impinging free shear layers, D. Rockwell & E. Naudascher.
Vortex interactions, P. G. Saffman & G. R. Baker.
Air flow and sound generation in musical wind instruments, N. H. Fletcher.
Finite amplitude baroclinic instability, J. E. Hart.
Ship boundary layers, L. Landweber & V. C. Patel.
Drop formation in a circular liquid jet, D. B. Bogy.
Rotating, self-gravitating masses, N. R. Lebovitz.
Mechanics of animal joints, V. C. Mow & W. M. Lai.
Numerical solution of compressible viscous flows, R. W. MacCormach & H. Lomax.
Wakes in stratified fluids, J.-T. Lin & Y.-H. Pao.
Internal waves in the ocean, C. Garrett & W. Munk.
On the spreading of liquids on solid surfaces: static and dynamic contact lines, E. B. Dussan V.
Geostrophic turbulence, P. B. Rhines.
The measurement of turbulence with the laser-Doppler anemometer, P. Buchhave, W. K. George & J. L. Lumley.
Similarity laws for constant-pressure and pressure-gradient turbulent wall flows, A. M. Yaglom.